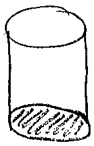
optimization

Test #1 Review Test (handout)

1. 

$SA = 2\pi r h + \pi r^2 = 3\pi$

$V = \pi r^2 h \leftarrow$ maximize

$$\frac{2\pi r h}{2\pi r} = \frac{3\pi - \pi r^2}{2\pi r} \rightarrow h = \frac{3\cancel{\pi}}{2\cancel{\pi} r} - \frac{\cancel{\pi} r^{\cancel{2}}}{2\cancel{\pi}\cancel{r}}$$

$$h = \frac{3}{2r} - \frac{r}{2}$$

$$V = \pi r^2\left(\frac{3}{2r} - \frac{1}{2}r\right) = \frac{3\pi}{2} r - \frac{\pi}{2} r^3$$

$$\frac{dv}{dt} = \frac{3\pi}{2} - \frac{3\pi}{2} r^2 = 0$$

$$\frac{\frac{3\pi}{2} = \frac{3\pi}{2} r^2}{3\pi/2}$$

$$1 = r^2$$

$$\boxed{r = 1 \text{ ft.}}$$

max !! ☺

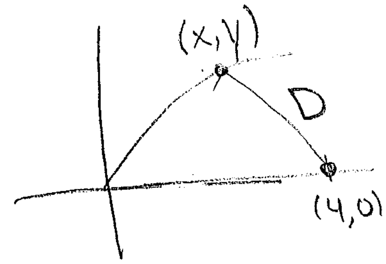
$\frac{dv}{dt}$: $+$ on $(0, 1)$, $-$ on $(1, \sqrt{3})$; marks at 0, 1, $\sqrt{3}$

$$h = \frac{3}{2r} - \frac{r}{2}\Big|_{r=1} \Rightarrow \boxed{h = 1 \text{ ft.}}$$

2. $y = \sqrt{x}$ Distance from (x, y) to $(4, 0) \leftarrow$ minimize

$$D = \sqrt{(x-4)^2 + (y-0)^2} = \sqrt{x^2 - 8x + 16 + (\sqrt{x})^2}$$

$$= \sqrt{x^2 - 7x + 16}$$



$$D' = \frac{1}{2}(x^2 - 7x + 16)^{-1/2}(2x - 7) = 0$$

$$2x - 7 = 0$$

$$x = 7/2 \rightarrow y = \sqrt{7/2}$$

min

D': $-$ before $7/2$, $+$ after $7/2$

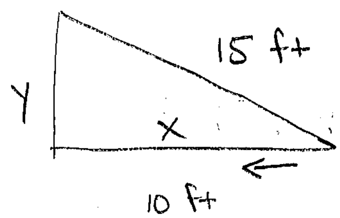
$$\boxed{\left(7/2, \sqrt{7/2}\right)}$$

Related Rates

1. $\frac{dV}{dt} = \frac{5cm^3}{min}$ $\quad \frac{dr}{dt} = ?\ cm/min$ $\quad$ ☆ $d = 20cm$
aka $r = 10cm$

$$V = \tfrac{4}{3}\pi r^3$$
$$\frac{dV}{dt} = 4\pi r^2 \, dr/dt$$
$$5 = 4\pi(10)^2 \frac{dr}{dt} \rightarrow \frac{dr}{dt} = \frac{5}{400\pi}$$
$$\boxed{\frac{dr}{dt} = \frac{1}{80\pi}\ cm/min}$$

2.



$\frac{dx}{dt} = -\frac{1}{4}\ ft/sec$ $\quad$ ☆ $t = 12\ sec$

$\frac{dy}{dt} = ?\ ft/sec$

In 12 sec the ladder moves 3ft so it will be 7ft from the wall

$$7^2 + y^2 = 15^2$$
$$y = \sqrt{176}$$

$$x^2 + y^2 = 15^2$$
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$
$$2(7)(-1/4) + 2(\sqrt{176})\frac{dy}{dt} = 0$$
$$2\sqrt{176}\frac{dy}{dt} = 14/4$$
$$\boxed{\frac{dy}{dt} = 0.132\ ft/sec}$$

Integration

☆ $u^{-1/3}$

1. $\int \frac{x^3}{(1+x^4)^{1/3}}dx \rightarrow \begin{matrix} u = 1+x^4 \\ du = 4x^3dx \end{matrix} \rightarrow \frac{1}{4}\int \frac{1}{u^{1/3}}du$

$$= \frac{1}{4}\cdot\frac{3}{2}u^{2/3} = \boxed{\frac{3}{8}(1+x^4)^{2/3} + C}$$

2. $\int \frac{\sin(\ln x)}{x}\,dx \rightarrow \begin{matrix} u = \ln x \\ du = \frac{1}{x}\,dx \end{matrix} \rightarrow \int \sin u \, du$

$= -\cos u \rightarrow \boxed{-\cos(\ln x) + c}$

Extrema / Points of Inflection

$f(x) = x^4 - 8x^2 \qquad f'(x) = 4x^3 - 16x$

$4x(x^2 - 4) = 0$

$x = \pm 2, 0$

$f'(x)$: $-$ | -2 | $+$ | 0 | $-$ | 2 | $+$

$\boxed{\text{maximum at } x = 0 \text{ and minimum at } x = \pm 2}$

$f''(x) = 12x^2 - 16 = 0$

$x = \pm \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$f''(x)$: $+$ | $\frac{-2\sqrt{3}}{3}$ | $-$ | $\frac{2\sqrt{3}}{3}$ | $+$

concave up $\left(-\infty, \frac{-2\sqrt{3}}{3}\right)$ $\left(\frac{2\sqrt{3}}{3}, \infty\right)$
concave down $\left(-2\sqrt{3}/3,\ 2\sqrt{3}/3\right)$

MVT, IVT, FTC

1. $\frac{f(2) - f(-4)}{2 - -4} = f'(c) \qquad \frac{-24 - 0}{6} = -4$

$-4 = 3x^2 - 16$

$12 = 3x^2$

$4 = x^2$

$x = \pm 2$

$\boxed{x = -2}$

2. $f(-5) = \frac{10}{-5-2} = \frac{10}{-7} < 0$

$f(5) = \frac{10}{5-2} = 10/3 > 0$

NO, Since $f(x)$ is <u>not</u> a continuous function the IVT doesn't apply!

3. $4x\sin(2x^2+7)$

Implicit Differentiation / Equ for a Line

1. $0 = (x^2)(3y^2y') + (y^3)(2x) + (x^3)(2yy') + (y^2)(3x^2)$

$y'(-3x^2y^2 - 2x^3y) = 2xy^3 + 3x^2y^2$

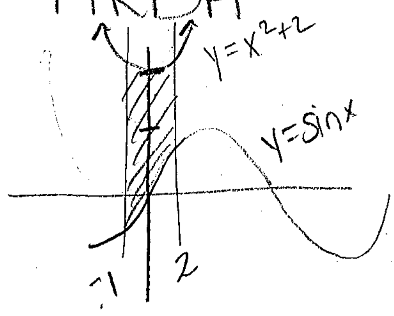
$$\boxed{y' = \frac{2xy^3+3x^2y^2}{-3x^2y^2-2x^3y}}$$

2. $f(x) = 5x^2+2 \qquad f(2) = 22$

$f'(x) = 10x\big|_{x=2} = 20$

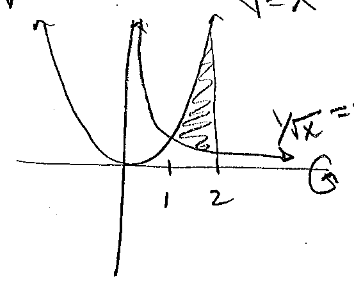
$$\boxed{y-22 = 20(x-2)}$$

AREA



$$\int_{-1}^{2} x^2+2-\sin x \, dx = \boxed{8.044}$$

Volume



Washer $R = x^2$

$r = \frac{1}{\sqrt{x}}$

$$\pi \int_1^2 x^4 - \frac{1}{x}\,dx = \boxed{17.3}$$

Arc Length

$$L = \int \sqrt{1 + \boxed{f'(x)}^2}\,dx$$

$y = \sqrt{x+2}$ $\qquad y' = \frac{1}{2}(x+2)^{-1/2}$

$$\int_1^7 \sqrt{1 + \tfrac{1}{4}(x+2)^{-1}} = \boxed{6.136}$$

Work

Slinky : $F = kd$ $\qquad$ Rocket: $F = k/d^2$

work to move it 10cm from 80cm to 3cm

$$\frac{40}{10} = \frac{K(10)}{10}$$

$K = 4$ N/cm

$$W = \int F\,dx$$

$$W = \int_{35}^{38} 4x\,dx = \boxed{438}$$

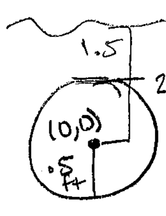
Fluid Force

$$W = P \int \text{slice} \cdot \text{distance}\ dy$$

P: density of liquid

slice: area or surface area

distance: distance the slice moves (or depth)



$x^2 + y^2 = .5^2$

$X = \sqrt{.25 - y^2}$

$$W = 64 \int_{-.5}^{.5} \left(2\sqrt{.25 - y^2}\right)(2 - y)\,dy$$

$$\boxed{= 100.531 \text{ lbs}}$$