

## AP Calculus BC – Test #1 – Review Problems – Chapters 1, 2, 3, 4, 5, and 7

### Optimization:

1. A container in the shape of a right circular cylinder with no top has surface area  $3\pi$  ft.<sup>2</sup>. What height  $h$  and base radius  $r$  will maximize the volume of the cylinder?
2. Find the point  $(x, y)$  on  $y = \sqrt{x}$  the graph of nearest the point  $(4, 0)$ .

### Related Rates:

1. Air is being pumped into a spherical balloon at a rate of  $5$  cm<sup>3</sup>/min. Determine the rate at which the radius of the balloon is increasing when the diameter of the balloon is  $20$  cm.
2. A  $15$  foot ladder is resting against the wall. The bottom is initially  $10$  feet away from the wall and is being pushed towards  $\frac{1}{4}$  the wall at a rate of ft/sec. How fast is the top of the ladder moving up the wall  $12$  seconds after we start pushing?

### Integration: Integrate the following

$$\int \frac{x^3}{(1+x^4)^{1/3}} dx$$

$$\int \frac{\sin(\ln x)}{x} dx$$

### Finding Extrema and Points of Inflection:

Find the relative extrema and points of inflection for  $f(x) = x^4 - 8x^2$

### Mean Value Theorem, Intermediate Value Theorem, and Fundamental Theorem of Calculus:

1. Let  $g(x) = x^3 - 16x$  and let  $c$  be the number that satisfies the MVT for  $g$  on the interval  $[-4, 2]$ . Find the value of  $c$ .
2. Consider the function  $f(x) = \frac{10}{x-2}$ . According to the intermediate value theorem, is there a solution to  $f(x) = 0$  for a value of  $x$  between  $-5$  and  $5$ ?
3. Evaluate  $\frac{d}{dx} \int_{-2}^{2x^2+7} \sin(x) dx$

### Implicit Differentiation/Equation for a Line

1. Assume that  $y$  is a function of  $x$ . Find  $y' = dy/dx$  for  $5 = x^2 y^3 + x^3 y^2$
2. Find the equation for a the line tangent to the curve  $f(x) = 5x^2 + 2$  at  $x=2$ .

### Area

Find the area between  $y = x^2 + 2$ ,  $y = \sin x$ ,  $x = -1$ , and  $x = 2$ .

### Volume (disk, washer, shell, cross-sections)

The region bounded by the curves  $y = \frac{1}{\sqrt{x}}$ ,  $y = x^2$  when the region from  $x = 1$  to  $x = 2$  is rotated around the  $x$ -axis. What is the volume of the generated solid?

### Arc Length

What is the length of  $y = \sqrt{x+2}$  for  $x$  such that  $1 \leq x \leq 7$

### Work (slinky/rocket)

A spring has a natural length of 20 cm. A 40 N force is required to stretch (and hold the spring) to a length of 30 cm. How much work is done in stretching the spring from 35 cm to 38 cm?

### Fluid Force – check out <https://www.sophia.org/tutorials/fluid-force-integral-calculation>

A vertical circular porthole in an observation ship has diameter 1 ft. It is placed so that the center of the of the porthole is 2 ft below the surface of the ocean. Calculate the fluid force on the porthole (seawater: 64.0 lbs per cubic ft).